Note Title 1/17/2012

Lecture 4

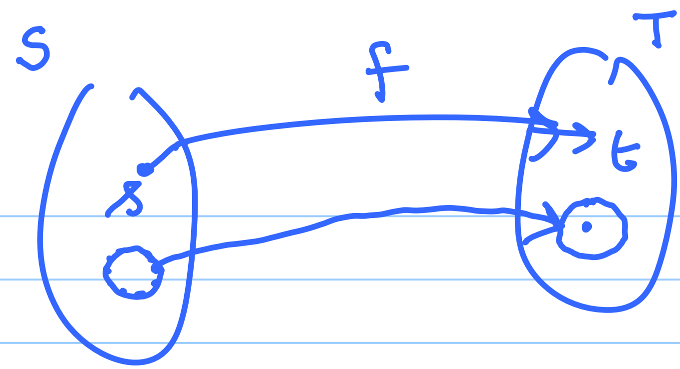
mappings, homeomorphism ||
diffeomorphism ||

characterize surface (manifold)

(1) Continuous map :

$$f(x) : R \to R$$

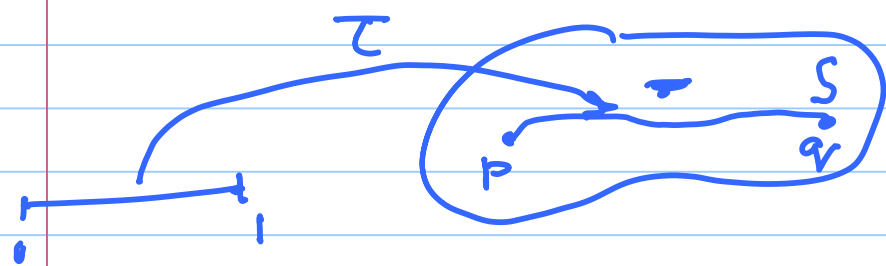
<u>pre-image</u> of an open set is an <u>open</u> <u>set</u>.



for $\varepsilon > 0$, $\exists\ \delta > 0$ such that

$d(s, x) < \delta \implies d(f(s), t) < \varepsilon$

12) path : a continuous map $\tau : [0,1] \to S$

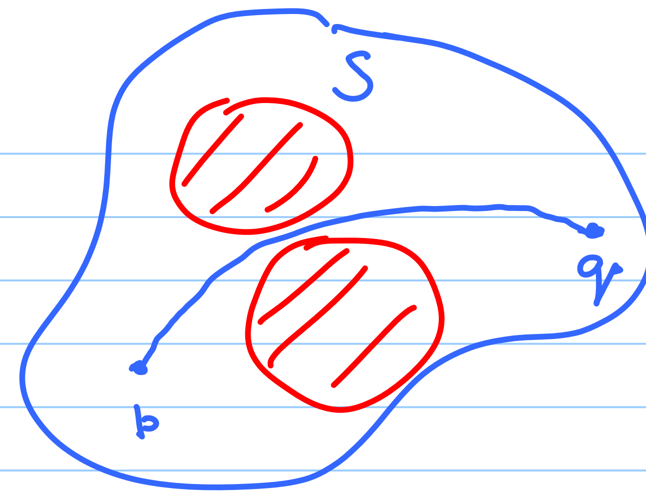


$\tau(0) = p$

$\tau(1) = q$

13) <u>path connectedness:</u>

A set S is path connected if given any two pts of the set, p, q, $\exists$ a path τ connecting p + q.



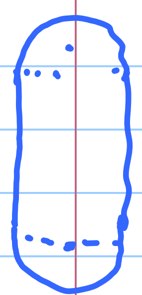
14) homeomorphism : X & Y are two open sets

is a mapping $\phi : X \longrightarrow Y$

1) bijective $\begin{cases} 1 \leftrightarrow 1 \\ \text{onto} \end{cases}$

2) Continuous

3) inverse or ϕ^{-1} is also continuous



15) diffeomorphism : (1) homeomorphism
(2) ϕ is smooth, and so is ϕ^{-1}

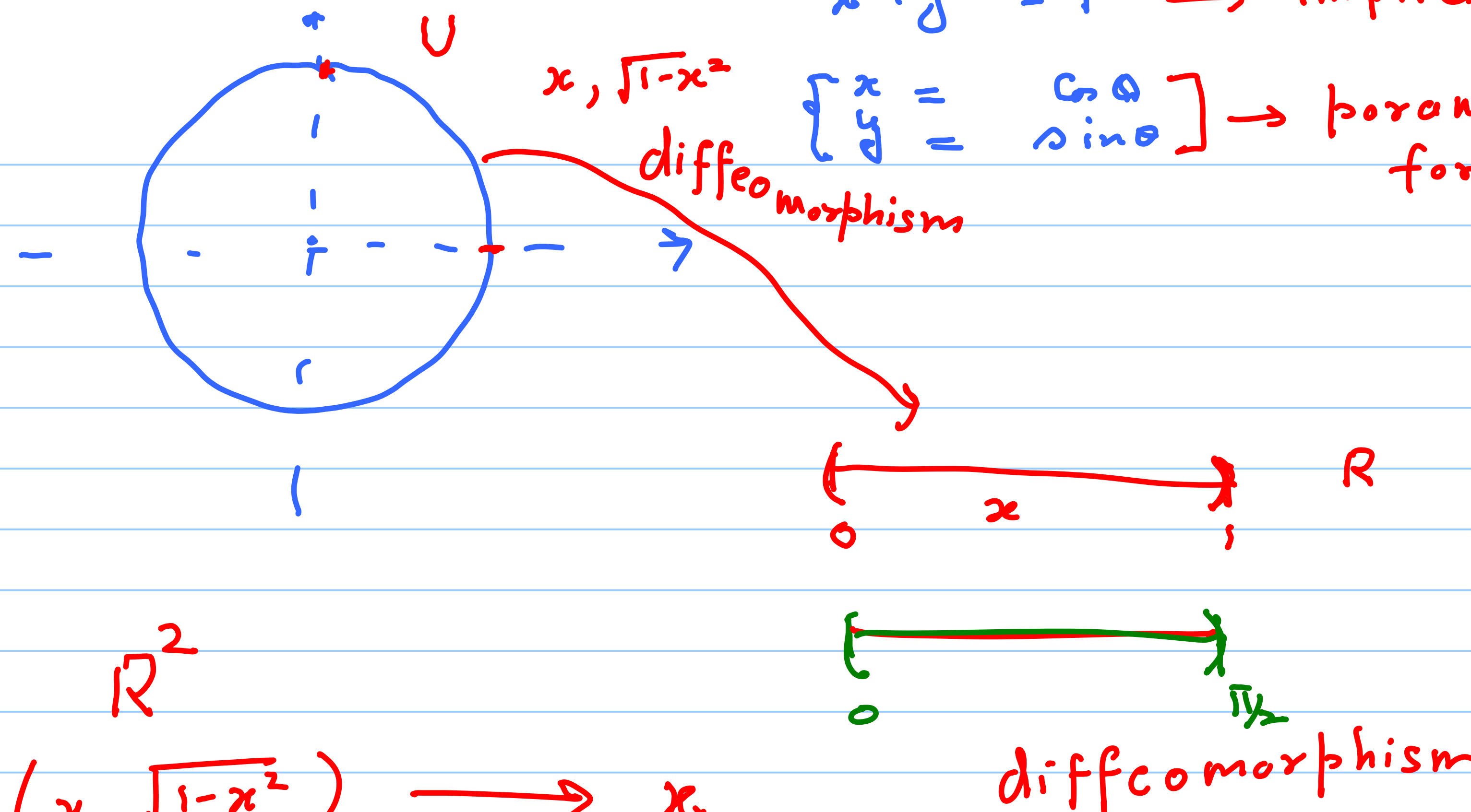
C^k continuous $\Longrightarrow$ k^{th} derivative is continuous

Smooth $\Rightarrow$ C^∞

Manifold $\longrightarrow$ formal notion of a surface

$x^2+y^2=1 \longrightarrow$ implicit

$\begin{bmatrix} x = \cos\theta \\ y = \sin\theta \end{bmatrix} \longrightarrow$ parametric form



R^2

$(x, \sqrt{1-x^2}) \longrightarrow x$

<u>Loose</u> " A set S is a manifold (differentiable) if

∃ a collection of diffeomorphisms whose domain covers S."

$U \subseteq S$

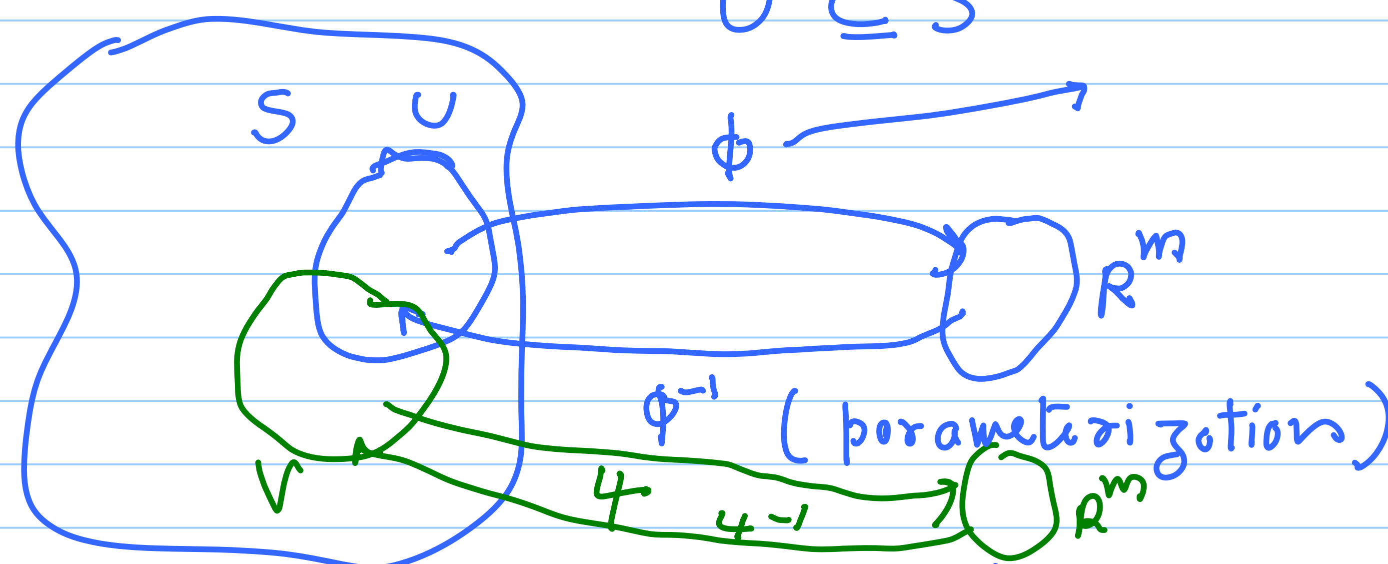


chart: (U, ϕ)

Manifold: UNION of charts

whose domain overlap.

U, V : $U \cap V$ = open set $\in S$

see example Fig 3.14 in Choset

Implicit func. theorem :

$$\underline{b} = \begin{pmatrix} x_{n+1} \\ \vdots \\ x_{n+m} \end{pmatrix}$$

$$\underline{f}(\underline{a}, \underline{b}) = \underline{0}$$

$$\underline{a} = \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix}$$

$$\underline{f} = \begin{pmatrix} f_1 \\ \vdots \\ f_m \end{pmatrix}$$

$\underline{a}_0, \underline{b}_0$

$$J = \begin{bmatrix} \frac{\partial f_1}{\partial x_{n+1}} & \cdots & \frac{\partial f_1}{\partial x_{n+m}} \\ \vdots & & \vdots \\ \frac{\partial f_m}{\partial x_{n+1}} & \cdots & \frac{\partial f_m}{\partial x_{n+m}} \end{bmatrix}$$

$$(x_{n+1}, \ldots x_{n+m}) : g(x_1, \ldots x_n)$$

Computational Geometry

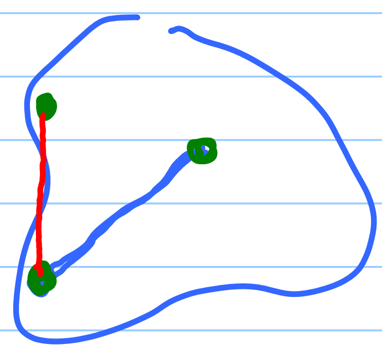
2D or 3D geometric entities

① Convex Set : S is Convex iff

$$\forall\ p, q \in S \Rightarrow \underbrace{\alpha p + (1-\alpha) q}_{\text{line of sight / visibility line}} \in S$$

$\alpha \in [0,1]$

line of sight
visibility line



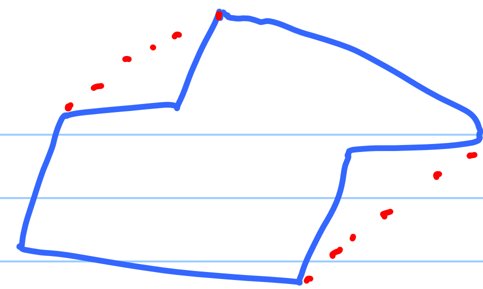
2) <u>Convex Combination :</u>

pts $x_1, x_2 \, . \, . \, . \, x_n$ are given

$$\sum_{i=1}^{n} \alpha_i x_i \qquad 0 \le \alpha_i \le 1$$
$$\sum \alpha_i = 1$$

What is the smallest convex set that encloses a given set S.

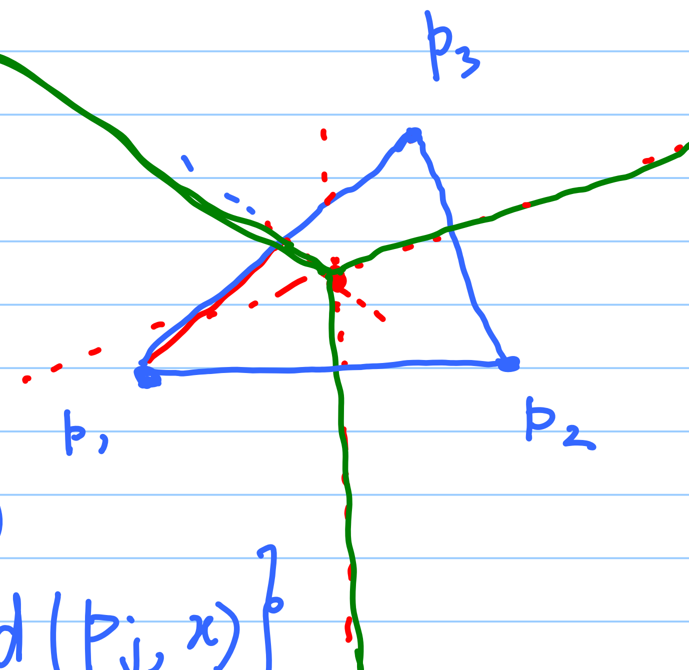
|| Convex Hull ||



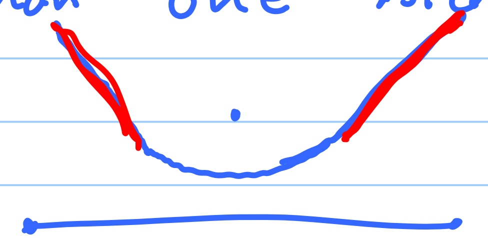
Voronoi Diagram:

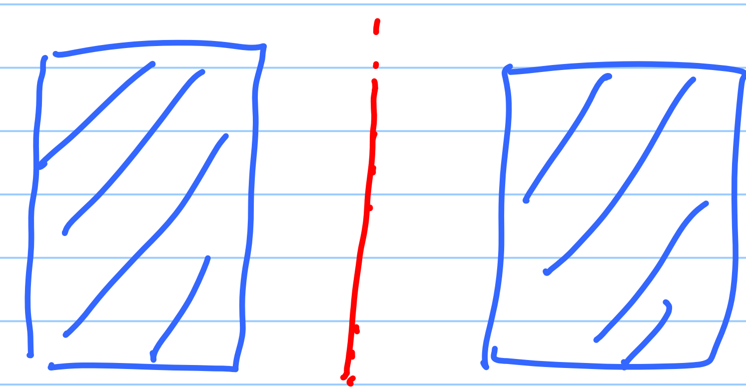
Voronoi region of p_i

$$V(p_i) = \{ x : d(p_i, x) \leq d(p_j, x) \ \forall j \neq i \}$$



Voronoi diagram : set of all pts x that ~~the~~ have more than one site closest to them.



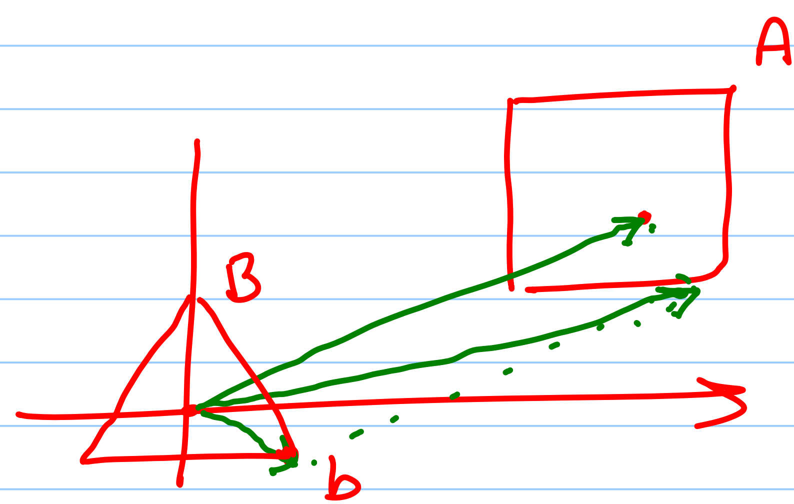


Delauney Triangulation: dual of Voronoi diag:

Connect sites (that share a voronoi segment) with an egde

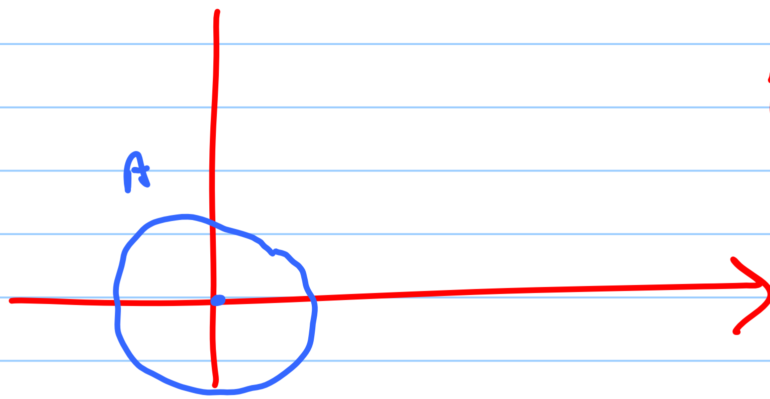
Minkowski sum :

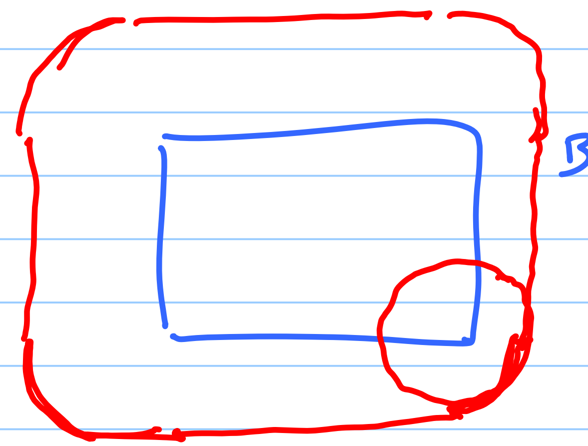
A & B are two sets in an affine space



$$A \oplus B = \{a + b,\ \forall a \in A,\ \forall b \in B\}$$

?





if A & B are convex compact sets : $\Pi(A) \to$ boundary of A

$$\Pi(A \oplus B) = \Pi(A) \oplus \Pi(B)$$